A General Framework for Optimization in Curbing Risk Contagion among Financial Networks

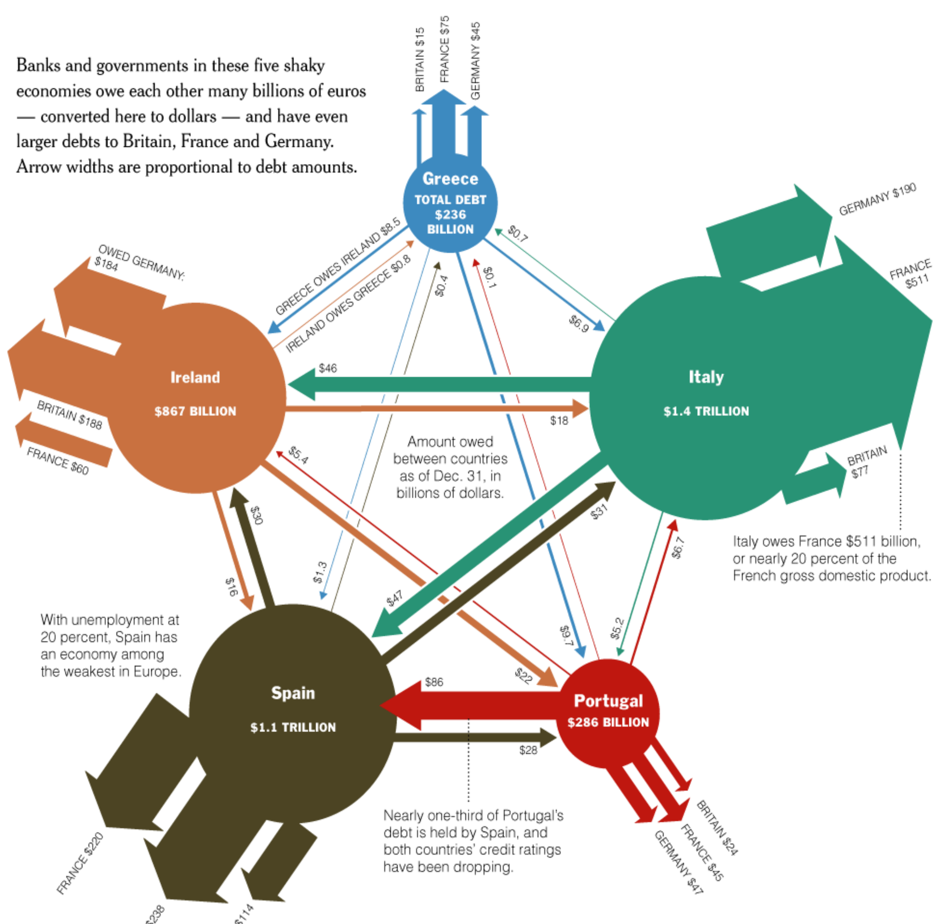
Xiang-Shen Ye[1, 2], Ruo-Bing Xue[1], Jian-Jun Gao[1], Tamer Başar[2], Xi-Ren Cao[1]

[1] Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China
[2] Coordinated Science Laboratory, University of Illinois, IL 61801, USA

★ RISK CONTAGION

- Financial institutions are interconnected by holding debt claims against each other.
- The interconnection is a key contributing factor to the worldwide financial crisis and the European sovereignty debt crisis.
- A default bank may cause its creditors to default, and the risk may be further propagated to up-stream institutes.

★ OBJECTIVES



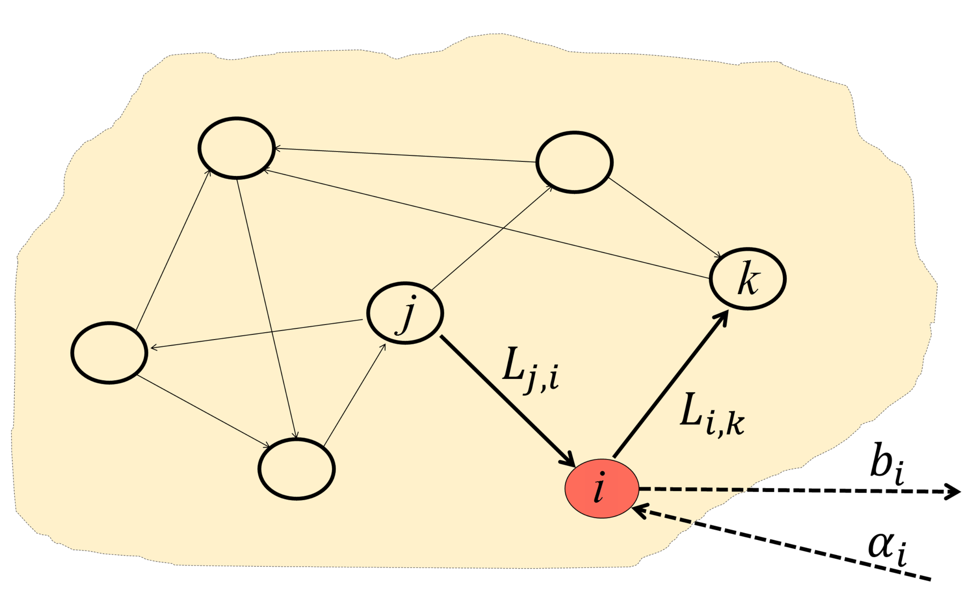
Source: Bill Marsh/The New York Times

How to curb risk contagion?
What is the role of a Central Bank?
What is the optimal liquidation scheme?

★ KEY WORDS

Direct-Comparison Based Optimization, Discrete Event Systems, Financial Network, Markov Decision Problems, Risk Contagion, Policy Iteration, Minimum Cost Flow

★ PROBLEM FORMULATION



Bank i has an obligation $L_{i,k}$ to bank k, a claim $L_{j,i}$ on bank j, outside assets α_i, and outside liabilities b_i

★ CONTRIBUTIONS

1. We develop a sensitivity based view of systemic risk modeling to characterize analytically how the mechanism of default liquidation affects the total wealth of the financial system.
2. We put forward two formulations, i.e., the Markov Decision Process model and the Min Cost Flow model.
3. We derive efficient iteration algorithms to this highly nonlinear problem.
4. We demonstrate the effectiveness of our approach for reducing systemic risk through examples.
5. Our work provides a new direction in curbing risk contagion in financial networks.
6. This work illustrates the advantages of the direct-comparison based approach, which originated in the field of Discrete Event Dynamic System, in nonlinear optimization problems.

★ MARKOV DECISION PROCESS MODEL

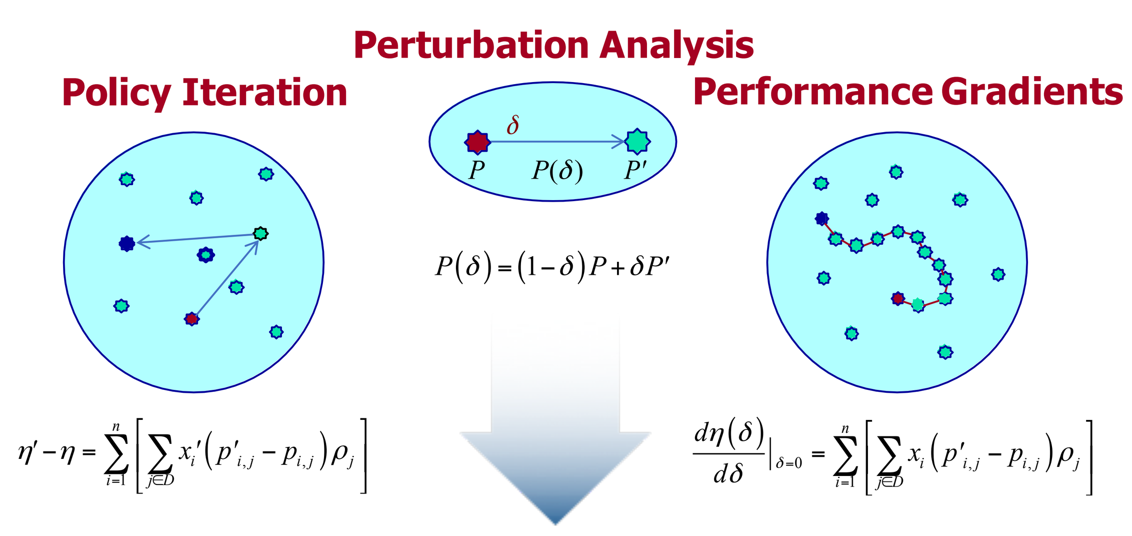
- We propose a possible role that the CB may take in curbing contagion: arbitrating the liquidation among banks in the system during the economic crisis and providing required compensation to achieve fairness.
- Allowing different liquidation schemes, we may reduce the system's loss and save banks from defaulting.
- This problem can be formulated as a nonlinear optimization problem with equilibrium constraints:

$$\max_P \left\{ \max_x |x| \right\}$$
$$s.t. \quad x = \min[l, \alpha + xP],$$
$$p_{i,i} = 0,\ p_{i,j} \ge 0,\ \sum_j p_{i,j} = 1 - b_i/l_i,$$
$$\forall i, j = 1, 2, \ldots, n, i \ne j.$$

- This is a Bi-Level (Leader-Follower) problem with:

Non-Convex Regions
Non-Linear Constraints
Large Dimensions

★ DIRECT-COMPARISON APPROACH



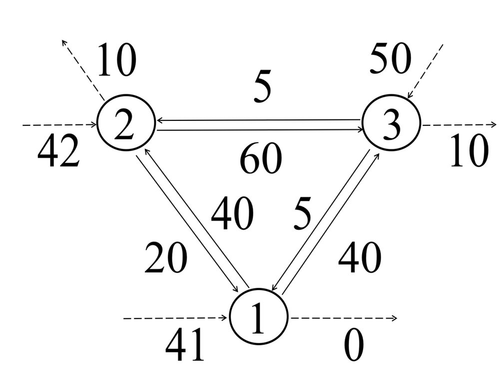
Policy Iteration and Gradient Based Algorithm
(Resembles a Markov Decision Problem)

- *Initialization*
- *Policy Evaluation*
- *Policy Update*
- *Stopping Rule*

Find an optimal P^* to maximize η

★ NUMERICAL EXAMPLES

- *3-Bank Example*

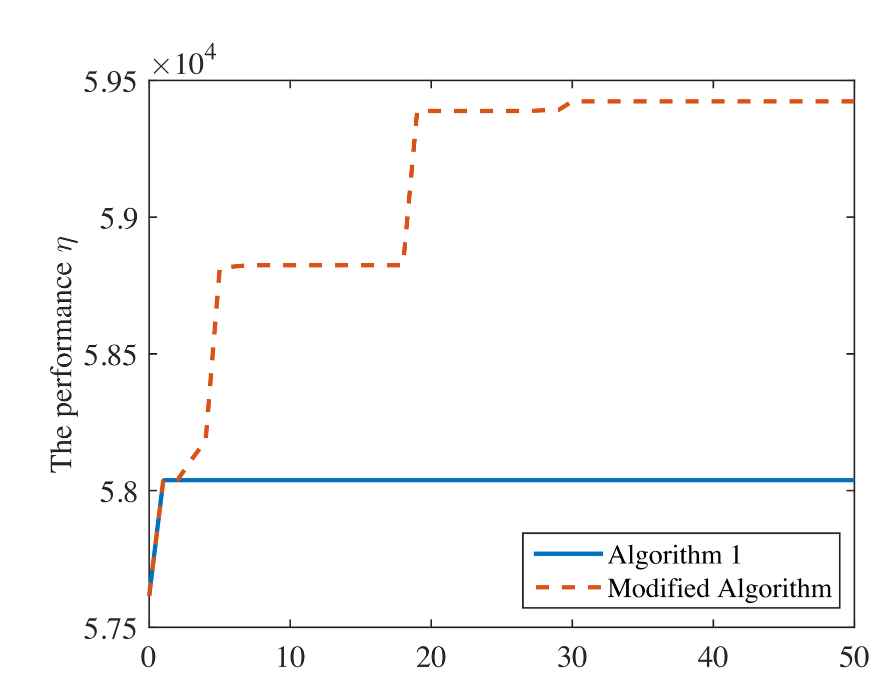


- Default Set

  D={1, 2}, D'={∅}
- Performance

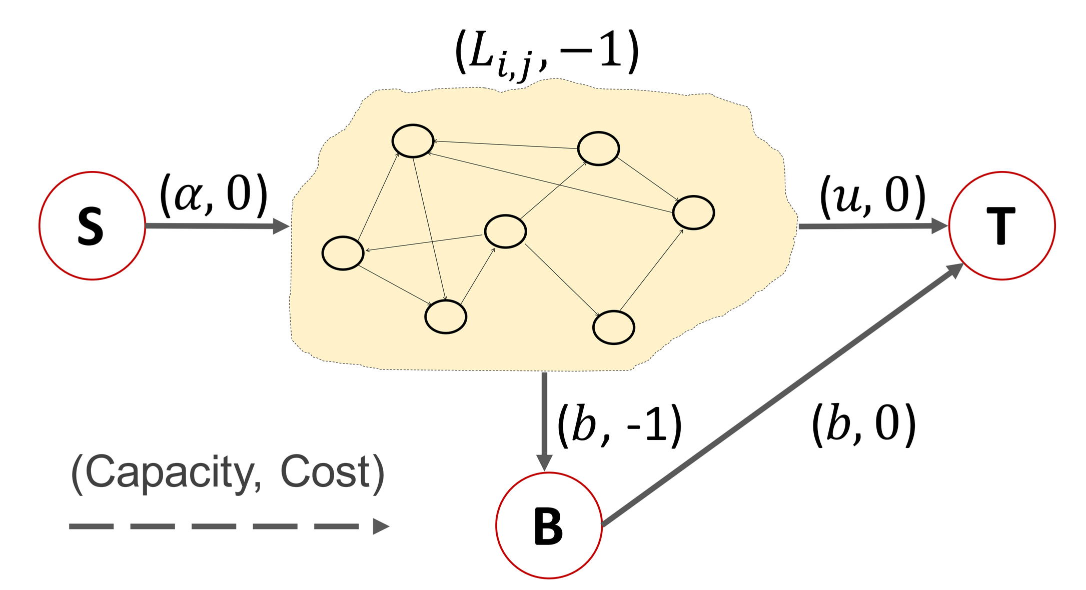
  η=162.26, η'=190
- *Achieves the local as well as **GLOBAL** optimal;*
- *Reduces **100%** of the system's loss;*
- *By only **ONE** iteration.*

- *50-Bank Example*



- *This is achieved **ONLY** by improving the liquidation scheme;*
- ***NO ADDITIONAL MONEY** is needed from the CB;*
- *The Modified Algorithm is **MORE EFFICIENT**.*

★ MIN COST FLOW MODEL



S: source node, B: outside liability node, T: sink node

$$u_{iT} = \max[\alpha_i + \textstyle\sum_{j=1}^n L_{j,i} - \sum_{j=1}^n L_{i,j} - b_i, 0],\ i = 1, 2, \ldots, n$$

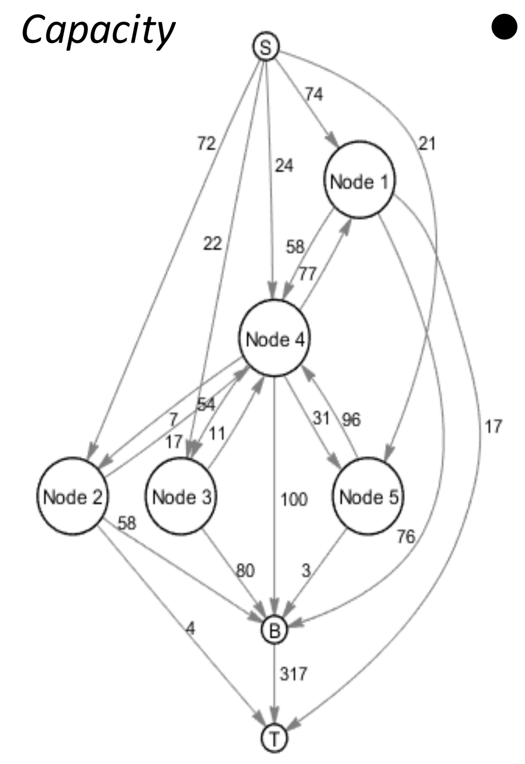
- We can formulate this system into a Min Cost Flow model shown as the above network ($G = (N, A)$).
- The system should satisfy the following conditions:
  1) All data (cost, demand, and capacity) are integral;
  2) The flow of inside arc (i, j) satisfies: $X_{i,j} \le L_{i,j}$;
  3) The cost of inside arc (i, j) satisfies: $c_{i,j} = -1$;
  4) Total demand equals total supply, i.e., $\sum_{i \in N} d(i) = 0$.
- This problem can be formulated as a linear optimization problem with equilibrium constraints:

$$\min_{X_{i,j}} \sum_{(i,j)\in A} c_{i,j} X_{i,j}$$
$$s.t. \quad 0 \le X_{i,j} \le u_{i,j},\ \forall (i,j) \in A,$$
$$\sum_{\{j:(i,j)\in A\}} X_{i,j} - \sum_{\{j:(j,i)\in A\}} X_{j,i} = d(i),\ \forall i \in N.$$

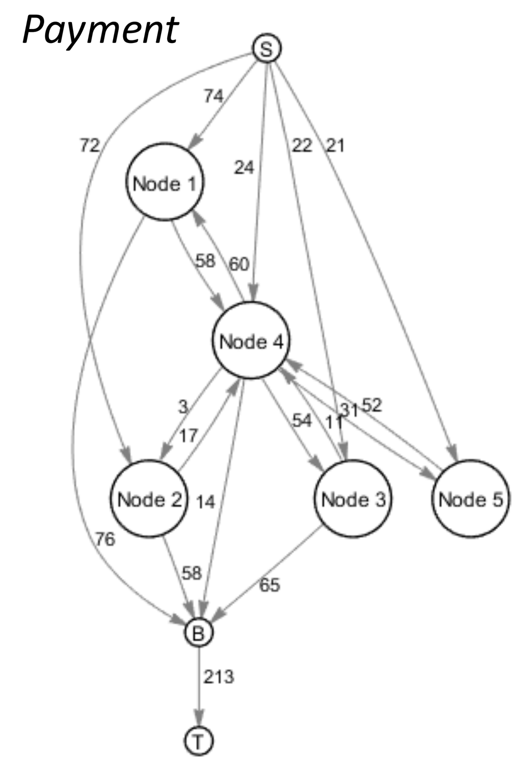
- The Min Cost Flow problem can be solved by the Network Simplex algorithm.

★ NUMERICAL EXAMPLES

- *Capacity*



- *Payment*



- Default Set

  D={1, 3, 4, 5}, D'={3, 4, 5}
- Performance

  η=400, η'=499
- *Achieves the local as well as **GLOBAL** optimal;*
- *Reduces **36.95%** of the system's loss;*
- *By only **10** iterations.*

★ REFERENCES

[1] X. R. Cao, *Stochastic Learning and Optimization: A Sensitivity-Based Approach*, Springer, 2007.

[2] X. S. Ye, R. B. Xue, J. J. Gao, and X. R. Cao, An iterative approach to reduce systemic risk among financial institutes, *IFAC'17*, Toulouse, France, July 9-14, 2017.

[3] X. S. Ye, R. B. Xue, J. J. Gao, and X. R. Cao, Optimization in curbing risk contagion among financial institutes, *Automatica*, accepted.

Xiang-Shen Ye
PhD Candidate
in Stochastic Optimization
Email: yesyjs@sjtu.edu.cn